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A COMPARATIVE STUDY OF DENOISING 1-D DATA USING WAVELET TRANSFORM

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ABSTARCT:
In this paper, discrete wavelet transform (DWT) for noise removal has been studied over four different types of synthetic data. These are blocks signal, bumps signal, doppler signal and heavy sine signal. These signal sets are first corrupted with white noise and then DWT is used for noise removal. The impact of different levels of decomposition along with the type of thresholding criterion, hard or soft, over output signal to noise ratio (SNR) and mean square error is presented. The waveforms of original and reconstructed signals for all levels of decomposition is also presented in the paper. All simulations are done in MATLAB.

Keywords: *Wavelets based denoising, Signal decomposition, Hard Soft Thresholding*

1. INTRODUCTION

Role of signals and systems is important in all aspects of modern technologies. However, a certain amount of unwanted signal i.e., noise is always introduced in digital signals. The noise is generally introduced while acquisition and transmission of data signals.

Introduction of noise in any signal degrades the quality of signal as well as the information content of the signal is lost to some extent. Hence, it is highly necessary to remove noise from the signal so that the information can be recovered from it. This type of signal restoration which is associated with an attempt of removal of noise from a signal is known as denoising. Noise is considered to have higher frequencies compared to signal. Hence, filtering was the most popular technique applied for denoising applications. Different linear and non-linear filters are proposed by researchers. Some examples of filters are mean filter, median filter, least mean square (LMS) filter, wiener filter, etc. Research in this field has led to development of new denoising techniques such as optimal estimate methods, spectral subtraction methods, general and genetic matching pursuit methods. However, the newest approach in this field is denoising using wavelet transform method.

The traditional methods for denoising were based on the Fourier transformation and the short-time Fourier transformation [13] but the analysis effects of them were not good enough as the requirement was time frequency representation of signal. In Fourier transformation, the signal analysis is done completely in frequency domain. It can only analyze what frequency components exist in the signal. Hence, the time and frequency information could not be analyzed at the same time. The concept of short-time Fourier transformation (STFT) analysis is a bit different and comparatively better. It used the concept of windowing the signal i.e., analyzing only a small section of the signal at a time.

To overcome the resolution problem in traditional approaches, the wavelet transform was introduced. The wavelet transformation is a time-frequency signal analysis tool having characteristics of Multi-resolution Analysis (MRA) [12]. It has an ability to represent signal's partial characteristic in both time and frequency domain. Hence, it is a kind of time-frequency localization analysis method. Another characteristic of wavelet transform is that the width of window function is variable. The window function is known as mother wavelet (a signal with tiny oscillations and which does not lasts forever). Hence, the wavelet transform is translated and dilated version of a mother wavelet.

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The discrete wavelet transform and it's variants is based on dyadic decomposition of a input signal into approximation and detail coefficients. Based on the methodology employed in decomposition of signals, wavelet transform may be termed as Discrete Wavelet Transform (DWT) or Discrete Wave Packet Transform (DWPT) [11] [12]. In DWT, only the approximation coefficients at each level are further decomposed. In DWPT, detail coefficients along with approximation coefficients are decomposed. Discrete wavelet transform (DWT) is a powerful tool using multi-resolution analysis (MRA) to analyze a signal. It has been used in speech recognition systems, signal and image compression and denoising.

2. METHODOLOGY

The basic methodology is used in noise removal in wavelet domain is shown in Figure 1.

2.1. *Forward Transform*

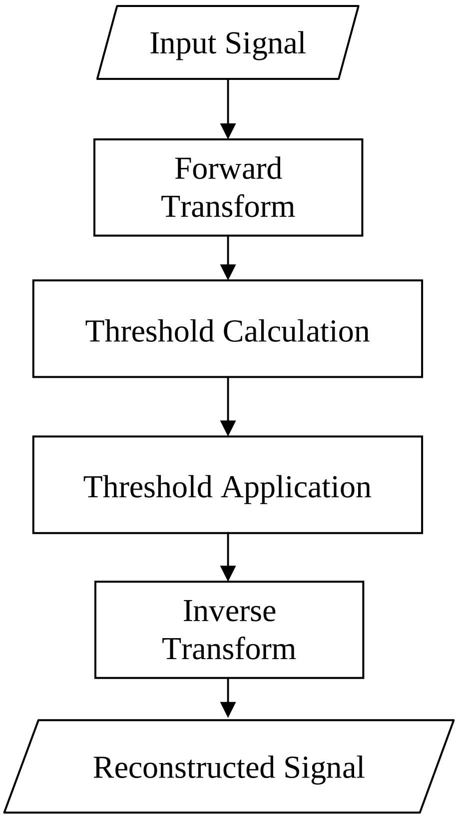

Fig. 1. Methodology

Input signal is transformed to wavelet domain using discrete wavelet transform.

2.2. *Threshold Calculation*

Removal of noise from a signal using wavelets is based on thresholding technique called wavelet thresholding. It is a non-linear technique. In this, some of the wavelet decomposed coefficients particularly detail coefficients which contain noise elements which are less than a predefined or dynamically computed threshold are made zero. Hence, this process is also known as wavelet shrinking [1]. Selecting an appropriate threshold is a topic of utmost importance as the efficiency of the algorithm entirely depend on its selection [2].

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Global and Level dependent Threshold

The calculation of threshold may or may not depend on the decomposition levels. If a threshold value is selected for each decomposition level , then the threshold is said to be level dependent threshold. However, if a same threshold value is calculated for coefficients at all levels, then the threshold is termed as global threshold [3] [14].

Various functions have been proposed to calculate an appropriate threshold value. Most accepted of those include VisuShrink and SureShrink. These are explained below.

VisuShrink

VisuShrink was introduced by Donoho [6]. It uses a threshold value t that is proportional to the standard deviation of the noise. It follows the hard thresholding rule. It is defined as:

$$t = \sigma\sqrt{2\log n} \ . \qquad (1)$$

σ^2 is the noise variance present in the signal and represents the signal size or number of samples. An estimate of the noise level was defined based on the median absolute deviation given by:

$$\hat{\sigma} = \frac{median\left(\left\{\left|d_{j-1,k}\right| : k = 0,1,\ldots,2^{j-1}-1\right\}\right)}{0.6745} \ . \qquad (2)$$

where $d_{j-1,k}$ corresponds to the detail coefficients in the wavelet transform.

VisuShrink can be viewed as general-purpose threshold selector which provide near optimal error properties. It also ensures that the estimates are as smooth as the true underlying functions.

Major drawbacks of VisuShrink include overly smoothed reconstructed signals as it removes too many coefficients, only applicable to signal corrupted with AWGN, and following of global thresholding scheme.

SureShrink

A threshold chooser based on Stein's Unbiased Risk Estimator (SURE) was proposed by Donoho and Johnstone [6] and is called as SureShrink. It is a combination of the universal threshold and the SURE threshold. This method specifies a threshold value for each resolution level in the wavelet transform which is referred to as level dependent thresholding. The goal of SureShrink is to minimize the mean squared error, defined as

$$\mathrm{MSE} = \frac{1}{n^2}\sum_{x,y=1}^{n}\left(z(x,y) - s(x,y)\right)^2 \ . \qquad (3)$$

where $z(x,y)$ is the estimate of the signal while $s(x,y)$ is the original signal without noise and n is the length of the signal. SureShrink suppresses noise by thresholding the empirical wavelet coefficients.

The SureShrink threshold t^* is defined as

$$t^* = \min(t, \sigma\sqrt{2\log n}) \ . \qquad (4)$$

where t denotes the value that minimizes Stein's Unbiased Risk Estimator, σ is the noise variance computed as in case of VisuShrink, and n is the length of the signal. The thresholding employed is adaptive, i.e., a threshold level is assigned to each dyadic resolution level by the principle of minimizing the Stein's Unbiased Risk Estimator for threshold estimates. It is smoothness adaptive which means that if the unknown function contains abrupt changes in the signal, the reconstructed signal also does.

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2.3. *Threshold Application*

After calculating a threshold value, a thresholding function $f(x)$ is required which is applied to threshold the coefficients. This function is dependent upon the threshold value t. There are two general categories to threshold function. They are hard threshold and soft threshold functions [8]. The hard threshold function [7] can be expressed as:

$$f_h(x) = \begin{cases} x & |x| \geq t \\ 0 & otherwise \end{cases}. \quad (5)$$

Hard threshold function is known to retain the sharp features of the signal however, it may mistake noise for true signal.
The soft threshold function [5] can be expressed as:

$$f_s(x) = \begin{cases} sign(x)\left[|x| - t\right] & |x| \geq t \\ 0 & otherwise \end{cases}. \quad (6)$$

Soft Threshold function is known to perform well in smooth regions but it over smooth sharp features. Hence the sharp characteristics of signal are lost.

Therefore, a new threshold function, hard-soft threshold was presented which unifies soft and hard threshold and may retain the chief signal features and at the same time, be possible to meet the smoothness requirements. The expression for hard-soft threshold is given below:

$$f_s(x) = \begin{cases} sign(x)\left[|x| - \alpha t\right] & |x| \geq t \\ 0 & otherwise \end{cases} \quad 0 \leq \alpha \leq 1. \quad (6)$$

The parameter α (scaling factor) defines the boundary between the hard and soft thresholding. If $\alpha = 1$, the function follow soft threshold while $\alpha = 0$ follow hard threshold. The choice of $0 < \alpha < 1$ tend to overcome the drawbacks of both hard and soft threshold functions.

The different threshold functions are shown in Figure 2. It can be inferred that hard threshold function keeps all the values of the signal for $|x| \geq t$ while completely removes all other values. Soft threshold function works similar to hard threshold except that it scales down the values. Hard soft threshold function provides a compromise between the other functions.

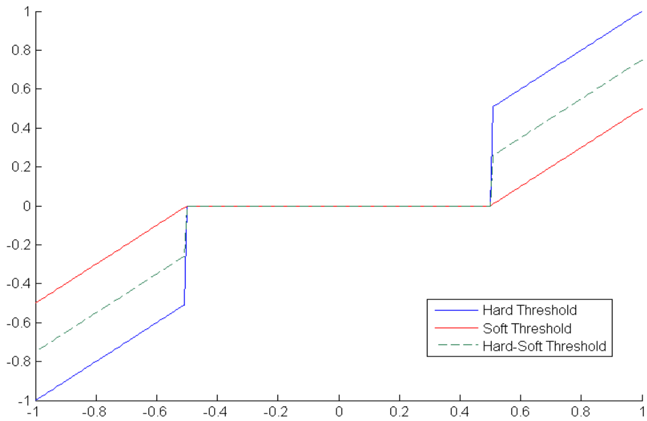

Fig. 2. Different threshold functions

3. SIMULATION SETUP

Four simulated signals are used for experimentation purpose. They are corrupted with additive white noise resulting in a noisy signal. Figure 3 show the four signals used, and Figure 4 show the corresponding noisy signals obtained.

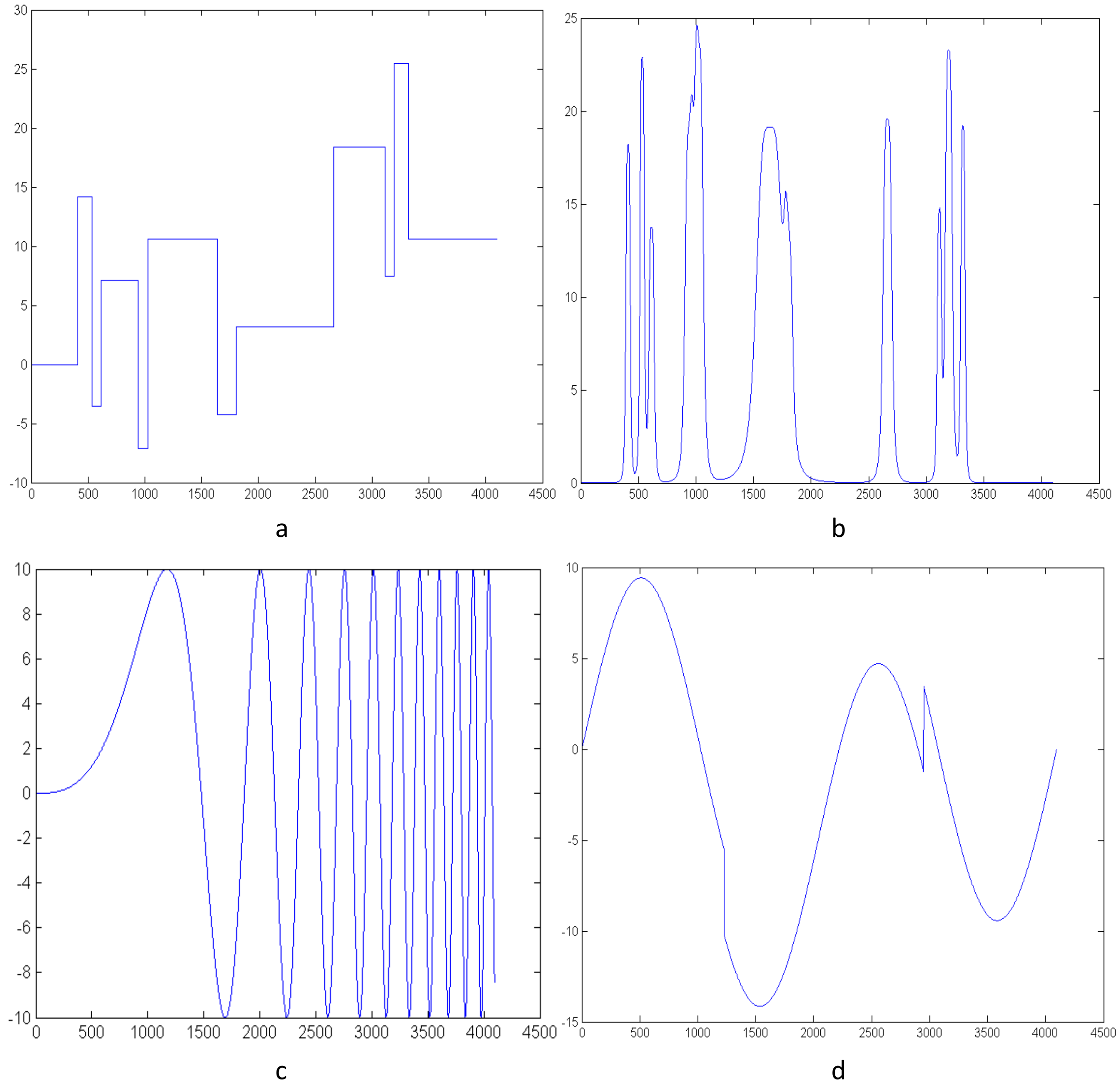

Fig. 3. Simulated Reference Signals (a) Blocks (b) Bumps (c) Doppler (d) Heavy Sine

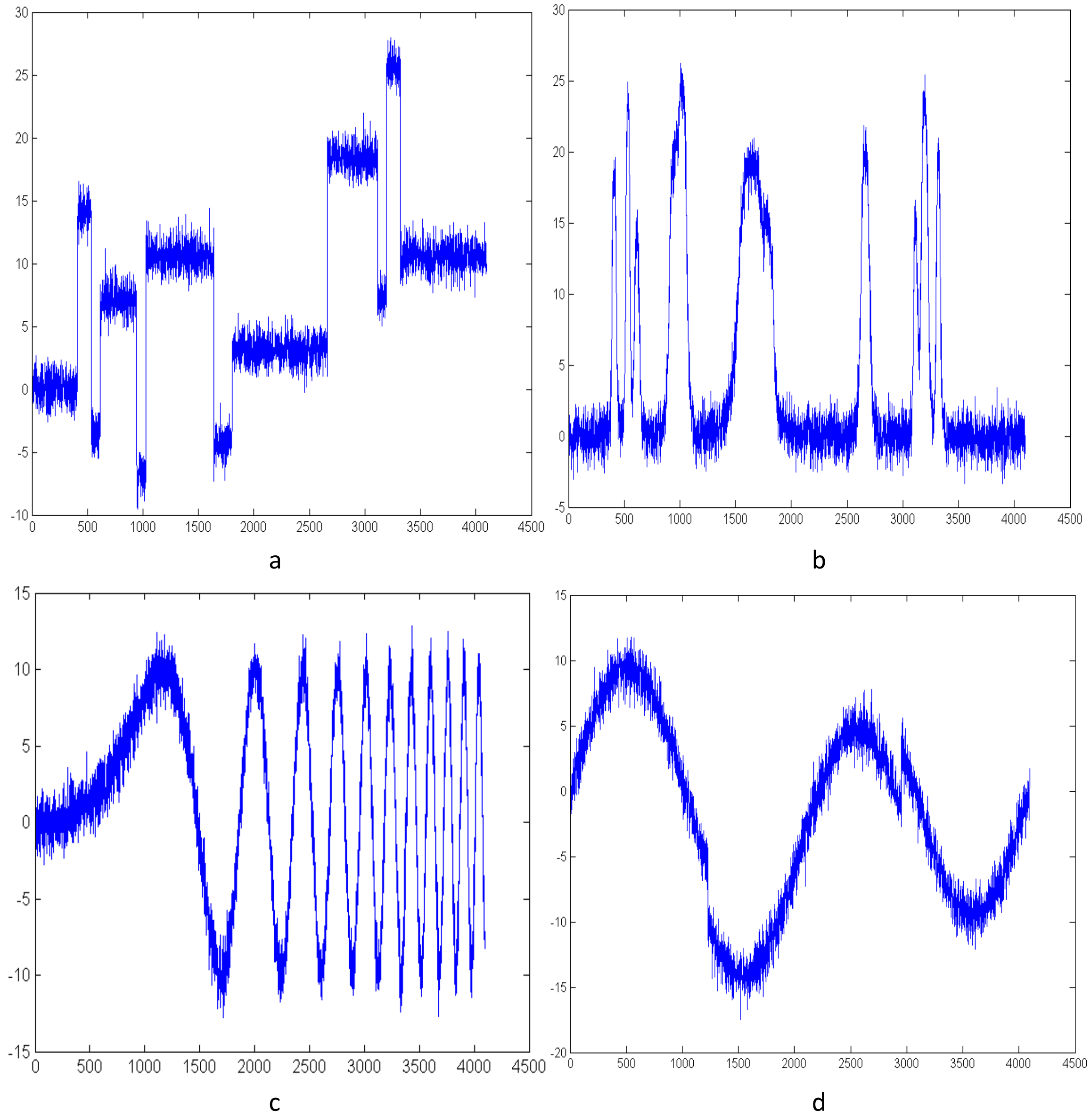

Fig. 4. Simulated Noisy Signals (a) Blocks (b) Bumps (c) Doppler (d) Heavy Sine

To compare the performance of proposed threshold selection algorithms and wavelet decomposition algorithms for one dimensional data, few parameters [9] [10] have to be computed for each case. The signal notations followed in this text are tabulated in Table 1.

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Table 1. Signal notations.

Notation	Signal
$s[n]$	Reference Signal
$w[n]$	Noise signal
$z[n]$	Noisy Signal
$y[n]$	Reconstructed Signal

Input Signal to Noise Ratio:
It is defined as

$$SNR_i = 10\log_{10}\left(\frac{\sum_{i=1}^{N} s^2[i]}{\sum_{i=1}^{N} w^2[i]}\right). \tag{7}$$

Output Signal to Noise Ratio:
It is defined as

$$SNR_o = 10\log_{10}\left(\frac{\sum_{i=1}^{N} y^2[i]}{\sum_{i=1}^{N}\{s[i]-y[i]\}^2}\right). \tag{8}$$

SNR_o should always be greater than SNR_i. Higher the SNR_o compared to SNR_i, better the algorithm.

Mean Square Error:
It is defined as

$$\varepsilon = \frac{1}{N}\sum_{i=1}^{N}\{s[i]-y[i]\}^2 . \tag{9}$$

Mean square error value should be minimum for better algorithm.

The wavelets used for evaluation purposes are haar for blocks and db8 for rest signals. The threshold selector used is universal threshold selector or VisuShrink. Different decomposition levels are also compared for performance evaluation which are 4, 5, 6, 7, 8 and 9. The results for soft and hard thresholding criterion are also compared.

4. RESULTS AND DISCUSSION

The results are compiled for each signal discussed in previous section one by one. First, the results of denoising noisy Blocks signal, Bumps signal, Doppler signal, Heavy Sine signal for all levels of decomposition are shown in figures 5, 6, 7 and 8.
The computed performance parameters are listed in Table 2.

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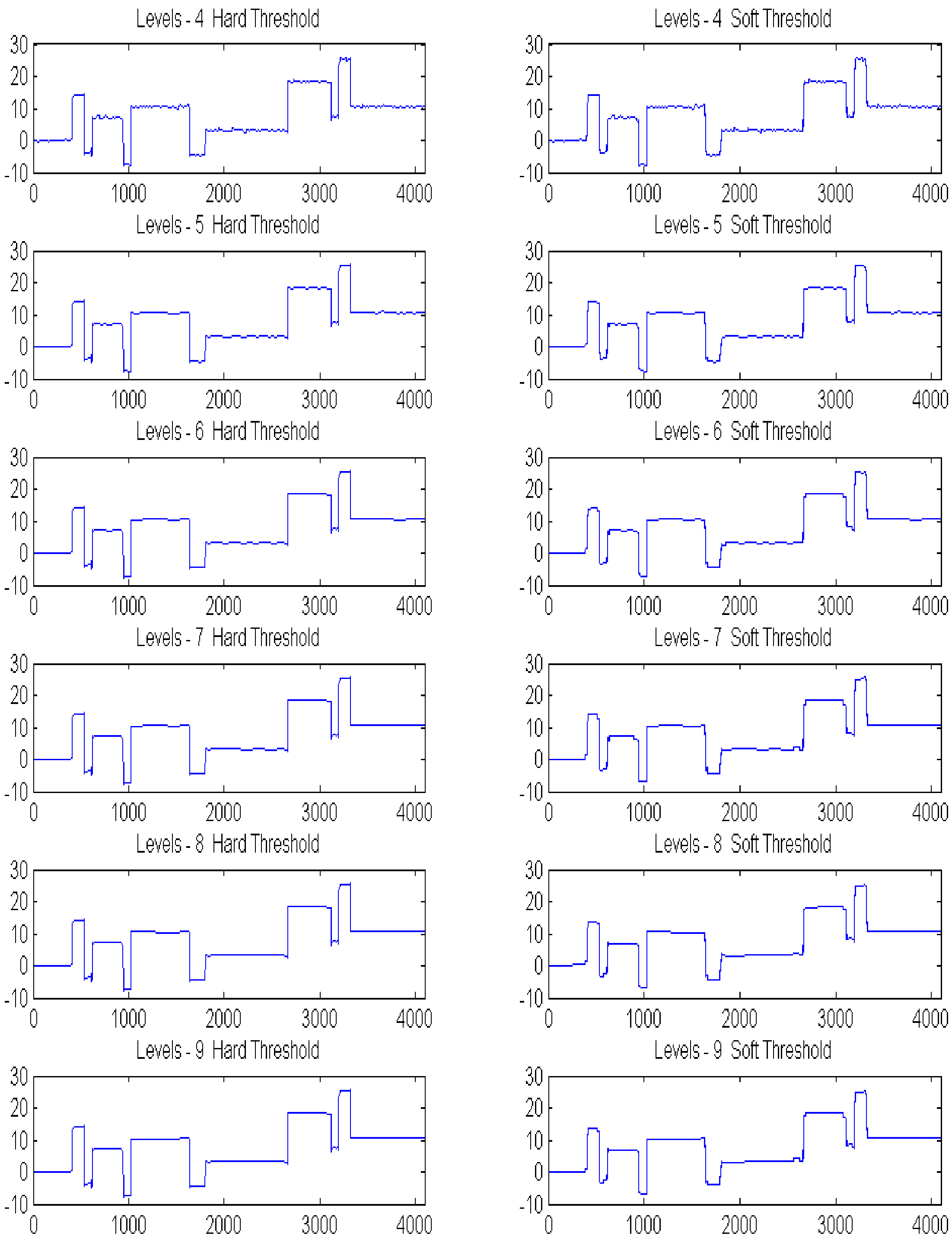

Fig. 5. Denoising results for Noisy Blocks signal.

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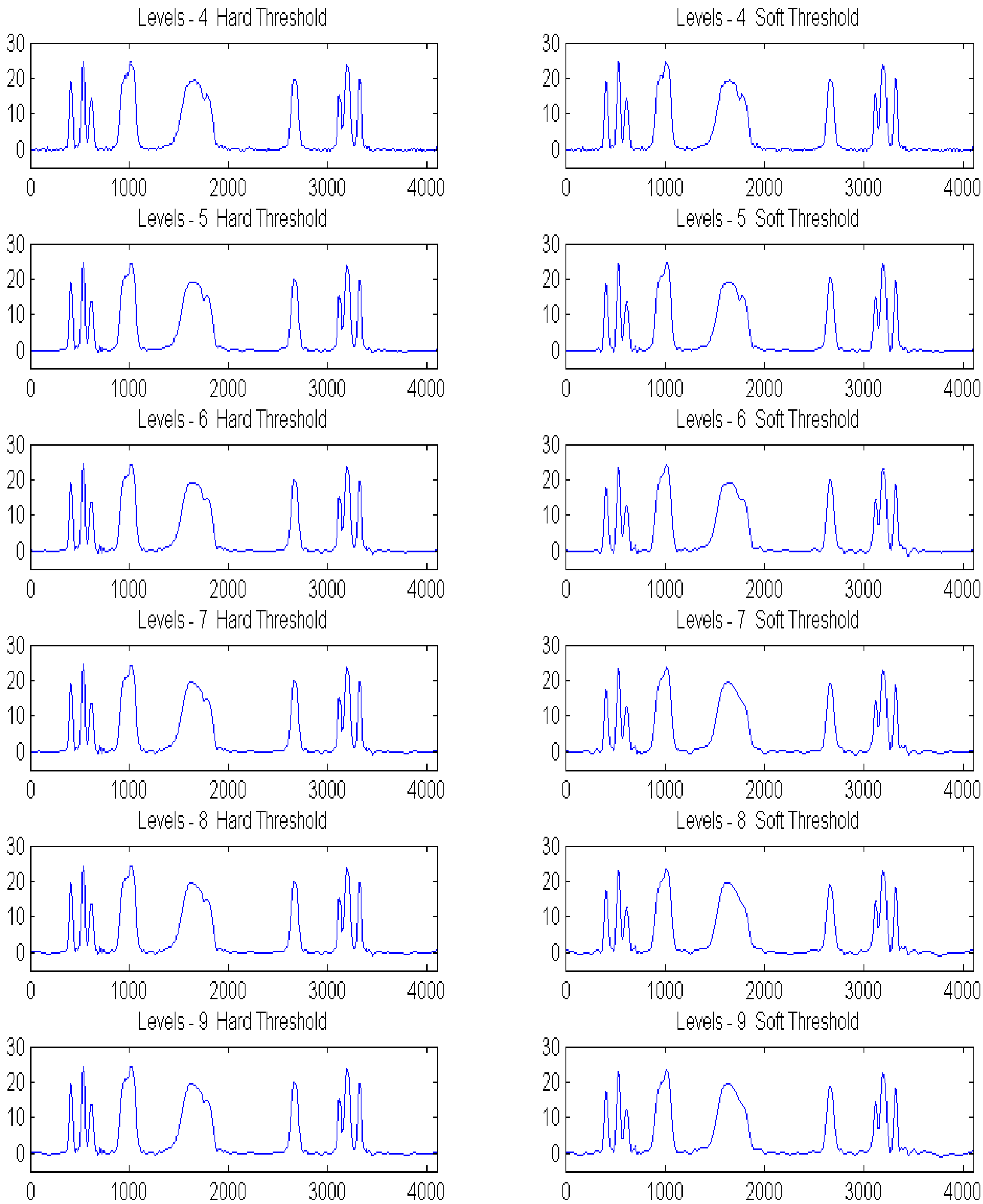

Fig. 6. Denoising results for Noisy Bumps signal.

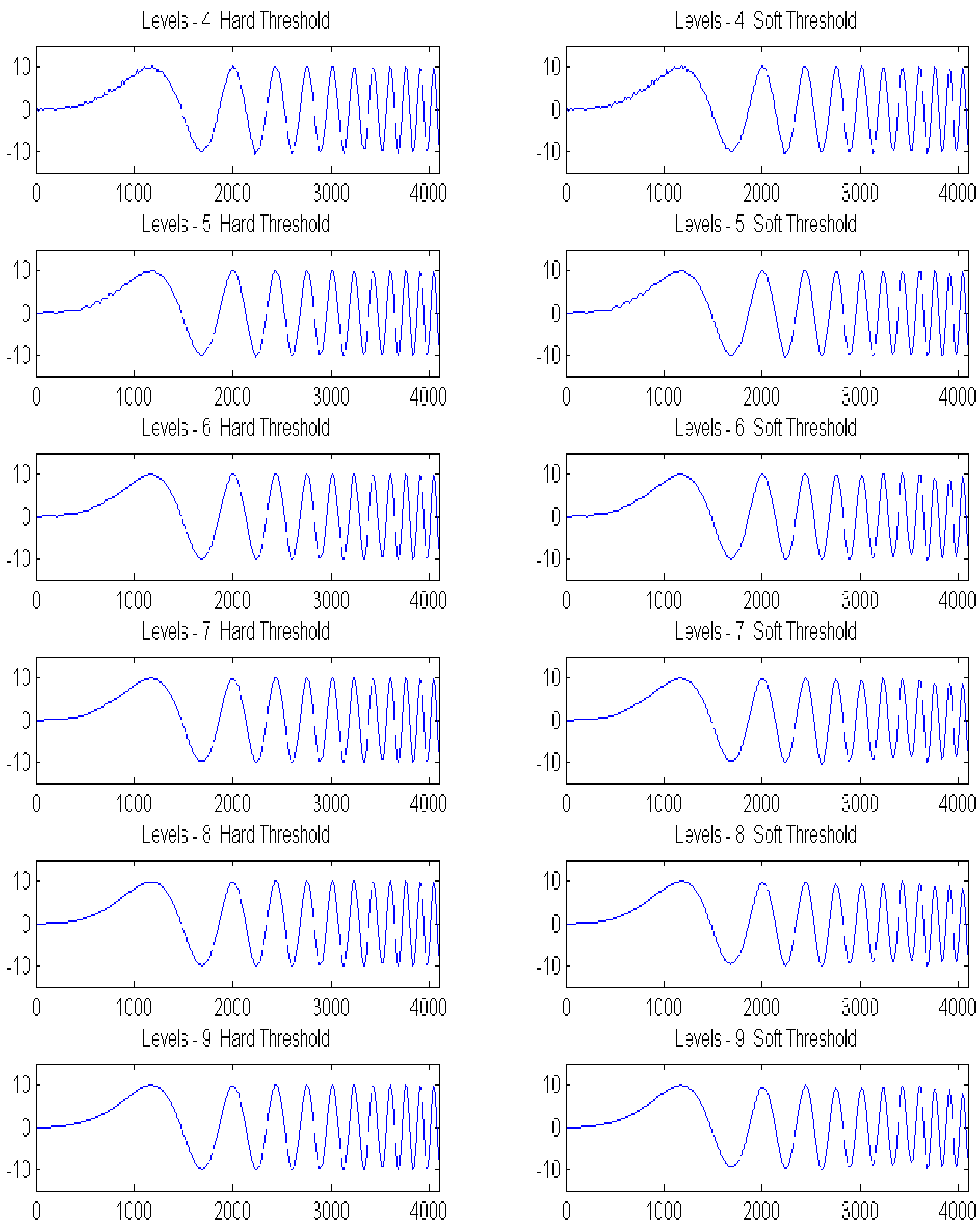

Fig. 7. Denoising results for Noisy Doppler signal.

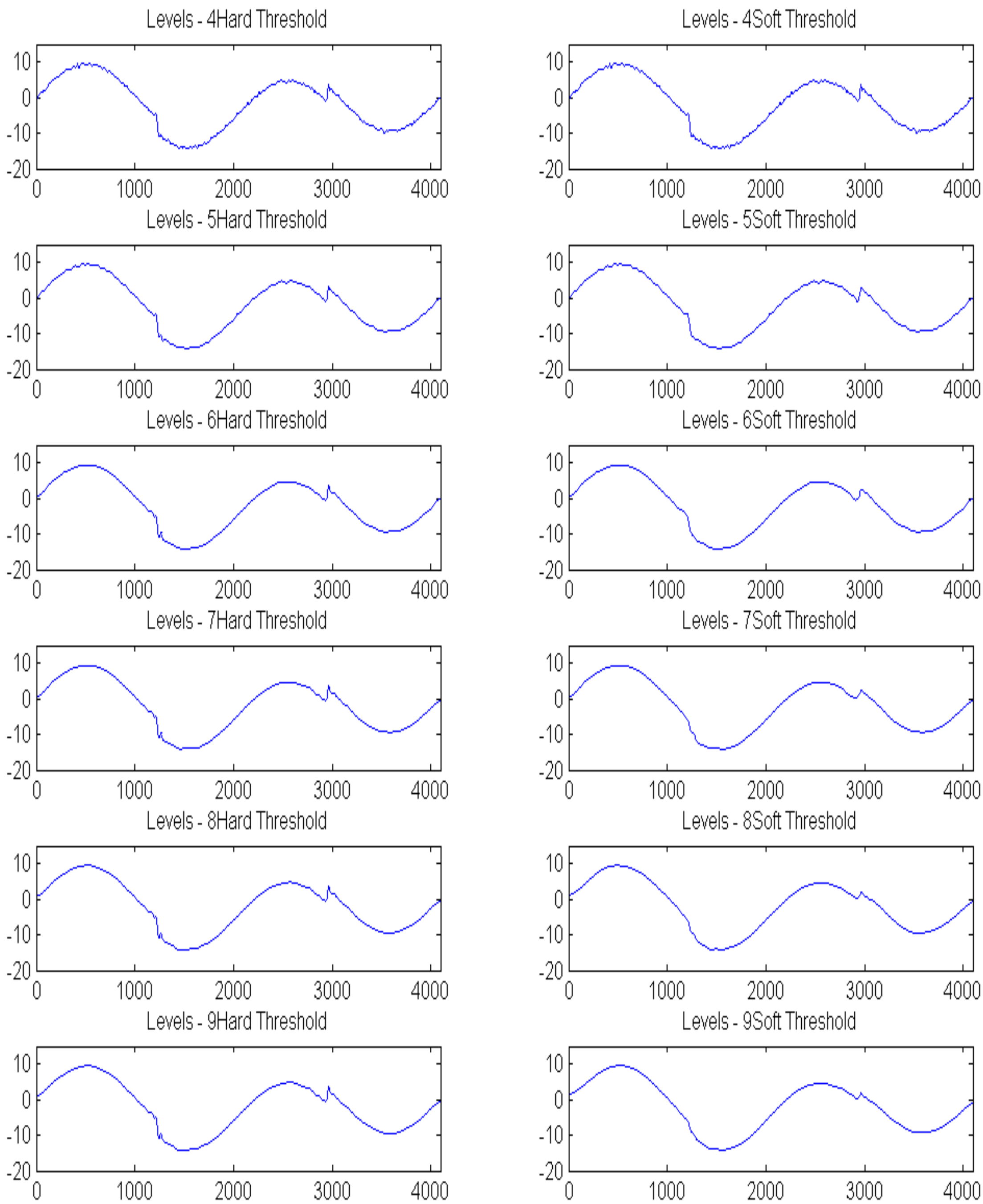

Fig. 8. Denoising results for Noisy Heavy Sine signal.

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Table 2. Performance Parameters.

Decomposition Levels	SNR_i	SNR_0		Mean Square Error	
		Soft Threshold	Hard Threshold	Soft Threshold	Hard Threshold
Noisy Blocks Signal					
4	20.53044	31.48765	27.18516	0.078079	0.20885
5	20.53044	33.44806	26.73679	0.049703	0.23074
6	20.53044	35.03504	26.36002	0.034485	0.250191
7	20.53044	35.79701	25.87709	0.028934	0.278555
8	20.53044	36.07692	25.47609	0.027127	0.302718
9	20.53044	36.13255	25.08213	0.026782	0.330014
Noisy Bumps Signal					
4	18.28755	28.67674	28.2855	0.090823	0.099367
5	18.28755	28.75359	25.60415	0.089177	0.183502
6	18.28755	27.90432	22.99783	0.108401	0.329285
7	18.28755	27.62304	21.68634	0.115641	0.436934
8	18.28755	27.41755	20.91398	0.121234	0.513478
9	18.28755	27.41336	20.54881	0.121347	0.548824
Noisy Doppler Signal					
4	16.2299	28.61423	28.61423	0.057789	0.057789
5	16.2299	31.22018	31.1995	0.031691	0.031841
6	16.2299	31.49929	26.86191	0.029703	0.085249
7	16.2299	30.83562	24.1856	0.034601	0.153208
8	16.2299	30.89928	22.90482	0.034099	0.201017
9	16.2299	30.77057	22.14459	0.035123	0.237039
Noisy Heavy Sin Signal					
4	17.17217	28.18741	28.18741	0.080296	0.080296
5	17.17217	30.15914	29.43562	0.050964	0.060184
6	17.17217	30.47433	29.10857	0.047375	0.064838
7	17.17217	30.3091	28.00843	0.049197	0.083425
8	17.17217	29.77172	26.48552	0.05566	0.118212
9	17.17217	29.70984	25.91715	0.056456	0.133977

Various conclusions can be drawn from the results. The impact of hard and soft threshold functions can be summarized as below:

(1) Soft Thresholding performs well in smooth regions.
(2) Soft Thresholding over smooths sharp features hence the sharp characteristics of signal are lost.
(3) Hard thresholding retains the sharp features of the signal.
(4) Hard thresholding may mistake noise for true signal.

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Also, on increasing the decomposition levels of the signal, the performance drops for soft threshold function while it rises up to a certain level and then drops a little and finally saturates for hard threshold function. MSE performance shown by the algorithm in almost all the cases is quite good which can be inferred from Table 2.

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